Midterm 2 - Review - Problems

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1 Derivatives

Problem 1

Find the derivatives of the following functions:

(a) $f(x) = \sqrt{\sin^{-1}(x)}$

(b) $f(x) = x^{\ln(x)}$

(c) $f(x) = x\cosh(x)$

(d) $f(x) = \ln(\ln(\ln(\ln(x))))$

(e) y' at $(2,1)$ if $x^2 + 2xy - y^2 + x = 9$

Problem 2

Show that the sum of the x and $y-$ intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c.

2 L'Hopital's rule

Problem 3

Evaluate the following limits

(a) $\lim_{x\to\infty} \left(\frac{\pi}{2} - \tan^{-1}(x)\right)^x$

(b) $\lim_{x\to 0} \frac{\sin(x)}{\sinh(x)+1}$

(c) $\lim_{x\to\infty} \frac{e^x - 1 - x}{x^2}$

(d) $\lim_{x\to 0^+} (\sin(x))^x$

3 Differential equations

Problem 4

Solve $y' = -2y$ with $y(3) = 2$

4 Linear approximation

Problem 5

Use linear approximations (or differentials) to approximate $\sqrt[4]{1.2}$. Is this an over- or underestimate?

5 Mean Value Theorem

Problem 6

Show that $x^5 - 6x + c$ has at **most** one solution in $[-1, 1]$

Problem 7

Is there a function f with $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all x?

6 Related rates

Problem 8

If the surface area of a ball is decreasing at a rate of 1 cm^2/min, how fast the volume decreasing when $r = 10cm$

Problem 9

[HARD] The minute hand on a watch is 8 mm long and the hour hand is 4 mm long. How fast is the distance between the tips of the hand changing at 1 o' clock?

7 Optimization problems

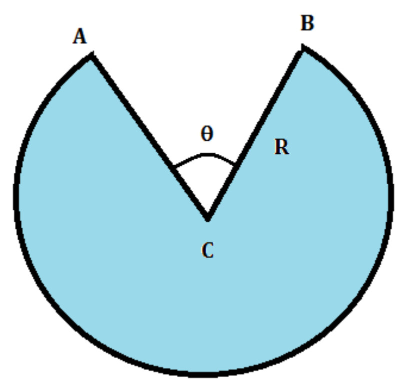
Problem 10

A rectangular playground is to be fenced off and divided in half by another fence parallel to one side of the playground. The area of each half is 600 sq. ft. Find the dimensions of the playground that will use the minimal amount of fencing.

Problem 11

[HARD] A cone-shaped drinking cup is made frm a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup (see picture below and the demonstration during the review session)

1A/Practice Exams/Drinking cup.png



8 Graphing

Problem 12

Graph $y = \frac{\sin(x)}{1+\cos(x)}$